DETERMINATION OF THE LOCAL ANGULAR RADIATION COEFFICIENTS IN CERTAIN TWO-BODY SYSTEMS

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A method is shown and formulas are derived by which local angular radiation coefficients can be determined in certain two-body systems where the configuration is arbitrary but one of the bodies is either a cylinder or a rectangular plate.

For a study and a numerical calculation of local energy characteristics in the case of radiative heat transfer, one must know the local angular radiation coefficients in the system. Formulas, graphs, and nomograms for finding the values of these coefficients are only available for a few combinations of body (surface) pairs [1, 2].

In this article the authors outline a method of determining the local angular radiation coefficients which is applicable to a two-body radiation system where one of the bodies is either a cylinder or a flat rectangular plate.

Calculations show that the angular coefficients for a cylinder or a plate radiating to other surfaces remain almost the same over a wide range of cylinder diameters or plate widths. On the other hand, the angular coefficients for a cylinder or a plate irradiated from another source are proportional to their respective transverse dimension. In view of this, it is possible to consider now a cylinder or a plate with an infinitesimally small transverse dimension, i.e., to treat each as a "radiating line" [3].

On the basis of certain concepts concerning a radiation field [4], the local angular coefficient φ_{21} of radiation from an arbitrary elementary surface 2 (Fig. 1) located in the radiation field of body 1 to that body 1 will be defined as the scalar product between the unit radiation vector $\bar{\varphi}$ and the unit normal vector $\bar{n}^0$ at the center of that area 2:

$$\varphi_{21} = (\bar{\varphi} \cdot \bar{n}^0), \tag{1}$$

where $\bar{\varphi} = \bar{E}/E_1^{int}$.

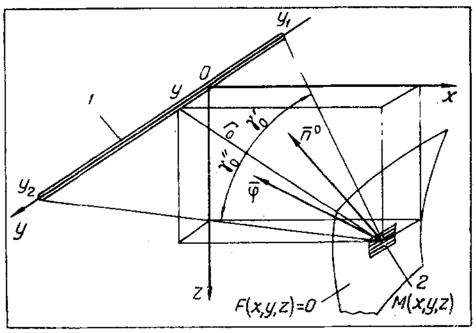

Fig. 1. Schematic diagram of the relative configuration between a linear radiator and an irradiated surface.

In order to derive analytical expressions for the local angular coefficients which would take into account the relative configuration between linear radiators and the area elements of an arbitrary surface, we introduce the rectangular system of coordinates shown in Fig. 1. The axis of the linear radiator 1 will be the y-axis and the photometric plane of this radiator will be the y0z plane of the coordinate system. The end points y_1 and y_2 of this linear radiator are located at arbitrary distances from the origin of coordinates.

Let the equation of the irradiated surface in this system of coordinates be

$$F(x, y, z) = 0. \tag{2}$$

The unit normal vector $\bar{n}^0$ at point M(x, y, z) of the elementary surface 2 is then defined in terms of direction cosines as follows:

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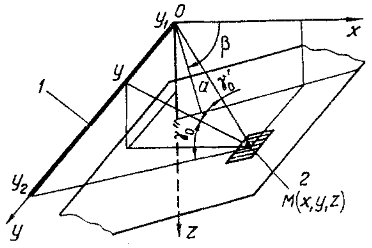
TABLE 1. Formulas for Calculating the Local Angular Radiation Coefficients φ_{21} in Certain Two-Body (Surface) Systems

Linear radiator parallel to plate

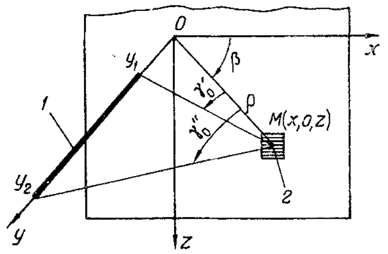


$$\varphi_{21} = \frac{sa}{\pi (x^2 + z^2)} \left| f(\gamma_0') - f(\gamma_0'') \right|,$$

for a cylinder $s = d,\ z = a,$

for a plate $s = \dfrac{t\,|z|}{\sqrt{x^2 + z^2}}$.

Linear radiator perpendicular to plate

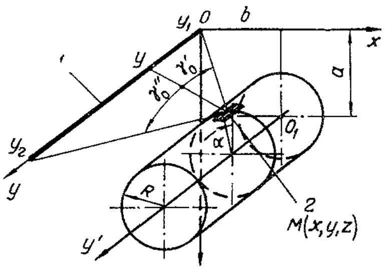


$$\varphi_{21} = \frac{s}{2\pi\rho} \left| \sin^2\gamma_0' - \sin^2\gamma_0'' \right|,$$

for a cylinder $s = d,$

for a plate $s = t \sin\beta.$

Linear radiator parallel to cylinder



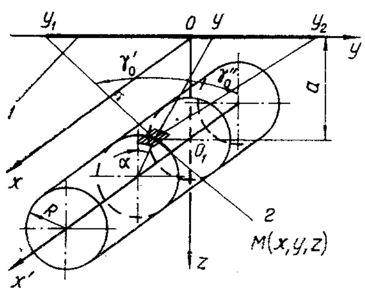
for a cylinder $s = d,\ b = 0,$

for a plate

$$s = \frac{|a - R\cos\alpha|}{\sqrt{a^2 + b^2 + R^2 - 2R(b\sin\alpha + a\cos\alpha)}},$$

$$\varphi_{21} = \frac{s(b\sin\alpha + a\cos\alpha - R)}{\pi\left[a^2 + b^2 + R^2 - 2R(b\sin\alpha + a\cos\alpha)\right]} \left| f(\gamma_0') - f(\gamma_0'') \right|.$$

Linear radiator skewed with cylinder at a 90° angle



for a cylinder $s = d,\ b = 0,$

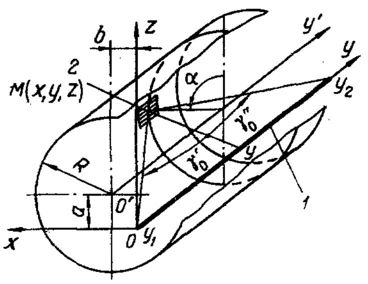
for a plate

$$s = \frac{t\,|a - R\cos\alpha|}{\sqrt{x^2 + (a - R\cos\alpha)^2}},$$

$$\varphi_{21} = \frac{s}{2\pi} \left\{ \frac{\sin\alpha \left| \sin^2\gamma_0' - \sin^2\gamma_0'' \right| \xi}{\sqrt{x^2 + (a - R\cos\alpha)^2}} + \frac{2(a - R\cos\alpha)\cos\alpha \left| f(\gamma_0') - f(\gamma_0'') \right|}{x^2 + (a - R\cos\alpha)^2} \right\}.$$

TABLE 1 (continued)

Linear radiator inside cylindrical cavity



for a cylinder $s = d,\ b = 0,$

for a plate

$$s = \frac{t\,|R\cos\alpha + a|}{\sqrt{a^2 + b^2 + R^2 + 2R\,(b\sin\alpha + a\cos\alpha)}},$$

$$\varphi_{21} = \frac{s\,(b\sin\alpha + a\cos\alpha + R)}{\pi\,[a^2 + b^2 + R^2 + 2R\,(b\sin\alpha + a\cos\alpha)]}\,|f(\gamma_0') - f(\gamma_0'')|.$$

$$\bar{n}^0 = \cos\alpha_x\bar{i} + \cos\alpha_y\bar{j} + \cos\alpha_z\bar{k}, \tag{3}$$

where

$$\cos\alpha_m = \frac{\dfrac{\partial F}{\partial m}}{\sqrt{\left(\dfrac{\partial F}{\partial x}\right)^2 + \left(\dfrac{\partial F}{\partial y}\right)^2 + \left(\dfrac{\partial F}{\partial z}\right)^2}}, \tag{4}$$
$$(m = x,\ y,\ z).$$

The unit radiation vector $\bar{\varphi}$ at point M(x, y, z) can also be written in terms of components parallel to the orthogonals $\bar{i}$, $\bar{j}$, $\bar{k}$:

$$\bar{\varphi} = \varphi_x\bar{i} + \varphi_y\bar{j} + \varphi_z\bar{k}. \tag{5}$$

Inserting (3) and (5) into (1), we obtain the following expression for the local angular radiation coefficient:

$$\varphi_{21} = \varphi_x\cos\alpha_x + \varphi_y\cos\alpha_y + \varphi_z\cos\alpha_z. \tag{6}$$

The values of φ_x, φ_y, φ_z can be found by an application of the general principles of field plotting to the case of radiation from a line source [3]. Vector $\bar{\varphi}$ at point M(x, y, z) will be tentatively directed toward the radiator. Then, inasmuch as the superposition property applies to radiant fluxes [1], the expressions for φ_x, φ_y, φ_z can be represented as functions of the coordinates and of an "apparent" transverse dimension s of the radiator surface (cylinder or plate):

$$\begin{aligned}\varphi_x &= -\frac{xs}{\pi r_0^2}\,|f(\gamma_0') - f(\gamma_0'')|,\\ \varphi_y &= \frac{s}{2\pi r_0}\,|\sin^2\gamma_0' - \sin^2\gamma_0''|\,\xi,\\ \varphi_z &= -\frac{zs}{\pi r_0^2}\,|f(\gamma_0') - f(\gamma_0'')|,\end{aligned} \tag{7}$$

where

$$r_0 = \sqrt{x^2 + z^2}; \tag{8}$$

$$\left.\begin{aligned}\gamma_0' &= \operatorname{arctg}\frac{y_1 - y}{r_0};\\ \gamma_0'' &= \operatorname{arctg}\frac{y_2 - y}{r_0};\end{aligned}\right\} \tag{9}$$

$$\left.\begin{aligned}f(\gamma_0') &= \frac{1}{4}(2\gamma_0' + \sin 2\gamma_0') = \frac{1}{2}\left[\operatorname{arctg}\frac{y_1 - y}{r_0} + \frac{(y_1 - y)\,r_0}{r_0^2 + (y_1 - y)^2}\right];\\ f(\gamma_0'') &= \frac{1}{4}(2\gamma_0'' + \sin 2\gamma_0'') = \frac{1}{2}\left[\operatorname{arctg}\frac{y_2 - y}{r_0} + \frac{(y_2 - y)\,r_0}{r_0^2 + (y_2 - y)^2}\right];\end{aligned}\right\} \tag{10}$$

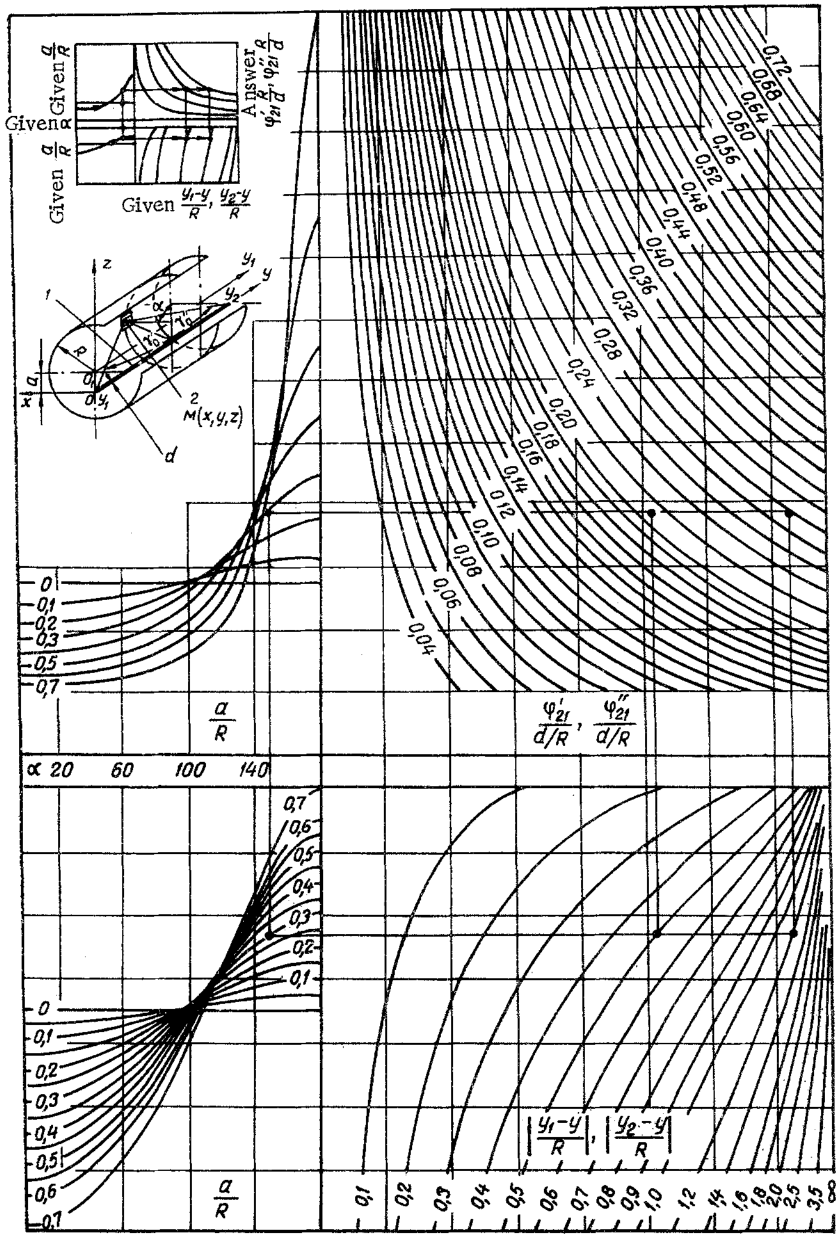


Fig. 2. Nomograms for determining the local angular coefficients of radiation from the inside surface of a cylindrical cavity to a cylinder inside it.

$$\sin^2\gamma_0' - \sin^2\gamma_0'' = \frac{(y_1 - y)^2}{r_0^2 + (y_1 - y)^2} - \frac{(y_2 - y)^2}{r_0^2 + (y_2 - y)^2}. \tag{11}$$

Here $\xi| = 1$ and has the same sign as whichever difference $(y_1 - y)$ or $(y_2 - y)$ is larger in magnitude.

If the radiator is a cylindrical surface with a small diameter d, then s = d in (7). If a narrow rectangular plate of width t replaces the linear radiator, then in (7)

$$s = \frac{t|z|}{r_0}.$$

The values of angles γ_0' and γ_0'' as well as their respective functions $f(\gamma_0')$ and $f(\gamma_0'')$ are determined from Eqs. (9)-(10) with due consideration of the signs of y, y_1, and y_2.

Generally, the area element 2 of a surface F(x, y, z) = 0 may be irradiated not from the entire linear radiator but from a part of it which, for nonconcave surface, lies in the half-space separated by the

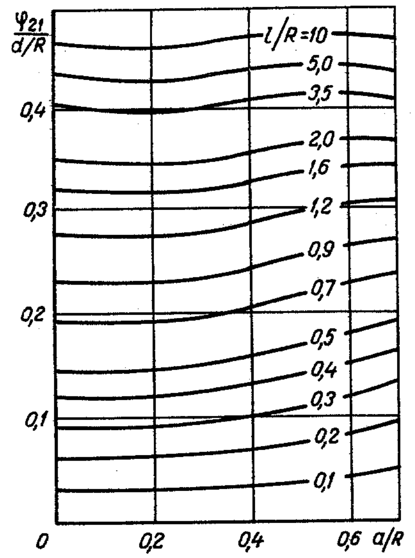


Fig. 3

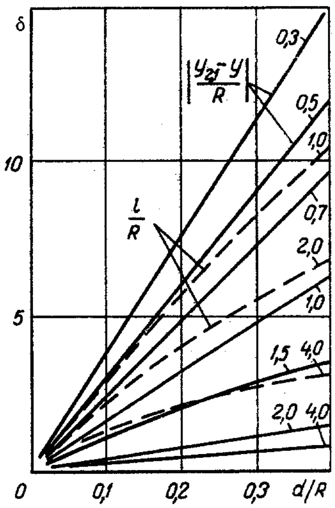


Fig. 4

Fig. 3. Mean angular coefficient φ_{21}^{m} of radiation from the inside surface of a cylindrical cavity to a cylinder of the same length inside it, as a function of the relative displacement between axes a/R and for various relative lengths l/R.

Fig. 4. Error δ (%) in calculating the local angular coefficients (solid lines) and the mean angular coefficients (dashed lines) of radiation from the inside surface of a cylindrical cavity to a concentric cylinder inside it, as a function of the geometrical characteristics of the system (d/R, $|(y_{21}-y)/R|$, l/R).

plane tangent to the irradiated surface at the center of its area element 2. The coordinate of the "apparent" end $y_{1,2}$ of the linear radiator is the point where the y-axis pierces the plane tangent to the irradiated surface at M(x, y, z):

$$y_{1,2} = \frac{1}{\frac{\partial F}{\partial y}}\left(\frac{\partial F}{\partial x}x + \frac{\partial F}{\partial y}y + \frac{\partial F}{\partial z}z\right). \tag{12}$$

Into Eqs. (9)-(11) belong the values of y_1 and y_2 found from Eq. (12).

The formulas in Table 1 for calculating the local angular radiation coefficients have been derived by the preceding method for certain relative configurations between a linear radiator and an irradiated surface. The formulas are sufficiently simple and, therefore, suitable for practical use over a wide range of geometries. A comparison between the values obtained for these coefficients by the formulas in Table 1 and by some exact relations [5-7] shows that, already when the transverse dimension of the linear radiator is down to one fifth of its length and to one half of its distance from the irradiated surface, the error does not exceed 7-10%. The calculation error decreases fast as the transverse dimension becomes still smaller.

The mean (integral) angular radiation coefficients for these particular systems of bodies are calculated by integrating the local angular coefficients over the surface F(x, y, z) = 0.

Example. Let us consider a system of two cylindrical bodies, one cylinder with a diameter d located inside the cavity of the other cylinder with a diameter 2R. Let their axes be parallel but, generally, not coincident (Table 1, line 5). Let the distance between the axes be a. The inner cylinder 1 will be treated as a linear radiator along the y-axis with the endpoints $y = y_1 = 0$ and $y = y_2$.

The equation of the outer cylinder surface is in our rectangular system of coordinates:

$$x^2 + (z-a)^2 - R^2 = 0. \tag{13}$$

The rectangular coordinates of a point on the surface of the outer cylinder can be defined in terms of cylindrical coordinates whose axis coincides with the y'-axis and whose angular coordinate α begins at the y0z plane in the counterclockwise direction:

$$x = R\sin\alpha, \quad y = y', \quad z = R\cos\alpha + a. \tag{14}$$

Having determined the direction cosines according to (4) and having inserted their values into (6) with (7) taken into account, we obtain the following expression for local angular coefficients of radiation φ_{21} from cylindrical surface 2 to cylinder 1:

$$\varphi_{21} = \frac{d(R + a\cos\alpha)}{\pi(a^2 + R^2 + 2Ra\cos\alpha)} \, |f(\gamma_0') - f(\gamma_0'')|. \tag{15}$$

When a = 0 and the inside cylinder is infinitely long, then formula (15) yields the well known exact expression for the local angular radiation coefficient [2], which in this case is equal to the mean angular radiation coefficient and is determined by the ratio of diameters:

$$\varphi_{21} = d/2R. \tag{16}$$

Nomograms of Eq. (15) have been plotted in Fig. 2 for determining the local angular coefficients of radiation from the inside surface of a cylindrical cavity to a cylinder located inside, as a function of the dimensionless coordinates $|(y_1-y)/R|$ and $|(y_2-y)/R|$, of the relative dimensions d/R, a/R, and of angle α. The values of φ_{21} are calculated as sums or differences of the angular coefficients φ_{21}' and φ_{21}'' for the segments y_1-y and y_2-y of the inner cylinder:

$$\varphi_{21} = \frac{d}{R}\left(\frac{\varphi_{21}'}{d/R} \pm \frac{\varphi_{21}''}{d/R}\right). \tag{17}$$

Thus, when α = 150°, a/R = 0.3, $(y_2-y)/R$ = 1.2, $(y_1-y)/R$ = 0.5, and d/R = 0.1, then φ_{21} = 0.1(0.297 + 0.210) = 0.0507.

The mean angular coefficients φ_{21}^{m} of radiation from a cylindrical cavity to an inner cylinder were determined by numerically integrating φ_{21} over the cavity surface. The results of calculations by this method are shown in Fig. 3, as functions of the relative displacement between axes a/R and of the relative cylinder length l/R.

The difference between the values of angular coefficients calculated by our method and by exact formulas [6, 7] respectively is shown in Fig. 4 for the given example but with coaxial cylinders (a = 0). According to the graphs, this difference is insignificant over a wide range of radiator geometries and the results may be used for a wide range of practical engineering applications.

NOTATION

$\bar{E}$	is the radiation vector of body 1;
E_1^{int}	is the intrinsic radiation intensity of body 1;
φ_x, φ_y, φ_z	are the components of the geometrical radiation vector along rectangular coordinates;
$r_0 = \sqrt{x^2 + z^2}$	is the shortest distance from point M(x, y, z) to linear radiator;
γ_0', γ_0''	are the angles subtending the two segments of the linear radiator from point M(x, y, z) on area element 2 of irradiated surface;
l	is the length of the cylinders;
x, y, z	are the space coordinates of point M.

LITERATURE CITED

1. A. G. Blokh, Fundamentals of Radiative Heat Transfer [in Russian], Gosénergoizdat (1962).
2. S. S. Kutateladze and V. M. Borishanskii, Textbook on Heat Transmission [in Russian], Gosénergoizdat (1959).
3. R. A. Sapozhnikov, Theoretical Photometry [in Russian], Izd. Énergiya, Leningrad (1967).
4. A. S. Nevskii, Radiative Heat Transfer in Furnaces and Ovens [in Russian], Metallurgizdat, Moscow (1959).
5. O. N. Favorskii and Ya. S. Kadaner, Problems of Heat Transfer in Space [in Russian], Izd. Vysshaya Shkola, Moscow (1967).
6. Yu. A. Surinov and S. V. Khorol'skii, Inzh.-Fiz. Zh., 14, No. 6 (1968).
7. F. M. Vafin, Teplofiz. Vys. Temp., 7, No. 4 (1969).